Computational tools in 3-dimensional topology
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I will demonstrate a wide variety of tools for studying Dehn surgery, hyperbolic geometry, Floer homology, foliations, character varieties, and Heegaard splittings, all of which can be used together via Python/SageMath via the computop/sage Docker image.
Dehn surgeon school: Nathan’s HW 1   Wednesday, July 17, 2019.

References and links for the second computational session, which demonstrated a wide variety
of tools for studying Dehn surgery, hyperbolic geometry, Floer homology, foliations, character
varieties, and Heegaard splittings, all of which can be used together via Python/SageMath via the
computop/sage Docker image.

- http://snappy.computop.org
- http://www.sagemath.org
- http://bitbucket.org/t3m/sagedocker
- http://doi.org/10.7910/DVN/LCYXPO
- http://github.com/bzhan/bfh_python
- http://regina-normal.github.io

1. Get SageMath and SnapPy working together on your laptop, for example using the computop
Docker image. Alternatively, from any of the physical ICERM terminals you can access it via
http://icerm2.icerm.brown.edu:8888.

2. You can get a knot of 14 or fewer crossings in SnapPy by doing:

   knots = snappy.HTLinkExteriors(cusps=1)
   E = knots.random()

Use verified computation as described here: http://snappy.computop.org/verify.html
to prove it is hyperbolic and to compute its volume to a provably correct 250 decimal places.
By Mostow rigidity this number is an invariant of the knot exterior and hence of the knot itself.
(There are a handful of non-hyperbolic links in this range, so you're very unlikely to pick one of
them and so be unable to complete this problem!)

3. Look at the documentation for HTLinkExteriors by typing

   ?snappy.HTLinkExteriors

   to figure out how to pick a random 10 crossing knot. Download the software of
   http://doi.org/10.7910/DVN/LCYXPO

   and use it to find coorientable taut/Reebless foliations on at least one Dehn surgery of your
   random knot.

4. Python programming practice:

   Use http://snappy.computop.org/spherogram.html to write a Python function to pro-
   duce a link projection of the \((a_1, a_2, \ldots, a_k)\) pretzel link. For the \((-2, 3, 7)\) pretzel knot, write
   a procedure that searches for the two slopes of the two lens space Dehn surgeries discovered
   by Fintushel-Stern. Use Regina to determine which lens spaces these are. Can you find lens
   space surgeries on other pretzel knots?

5. Look at the list of software that is part of the computop Docker image. See if you can compute
   something interesting with one of them.

6. The webpage http://computop.org lists a wide variety of computational tools in low-di-
   mensional topology. Find one that is relevant to your own work and try to get it working in
   your Docker container.